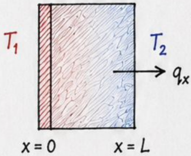
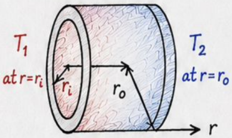
Fundamentals of HEAT TRANSFER

SUMMARY

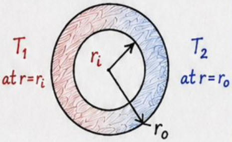
Plane Wall



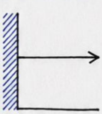
Cylindrical Wall



Spherical Wall

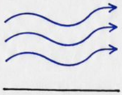


CONDUCTION



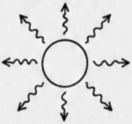
$$q_x = -k \frac{dT}{dx}$$

CONVECTION



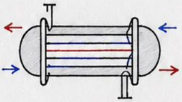
$$q'' = h\,(T_s - T_\infty)$$

RADIATION



$$q'' = \varepsilon \sigma \left(T_s^4 - T_{sur}^4\right)$$

HEAT EXCHANGERS



$$\dot{Q} = U A \,\Delta T_{lm}$$

By

Dr. Saeed J. Almalowi

Mechanical Engineering Department

E-mail: malouis@rcjy.edu.sa

X: @saeed_malowi

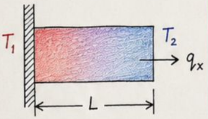
Heat Transfer – Summary

Heat flows from higher temperature to lower temperature.

Modes of Heat Transfer:

① Conduction

Heat transfer through a solid or stationary medium due to temperature gradient.



Fourier's Law:

$$\boxed{q_x = -kA\frac{dT}{dx}} \quad (W)$$

For plane wall (1-D, steady):

$$\boxed{q = kA\frac{(T_1 - T_2)}{L}}$$

k = thermal conductivity (W/m·K)

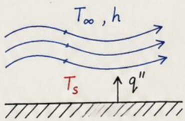
A = area normal to heat flow (m^2)

L = thickness (m)

$\frac{dT}{dx}$ = temperature gradient (K/m)

② Convection

Heat transfer between a surface and a moving fluid. (involves conduction + fluid motion)



Newton's Law of Cooling:

$$\boxed{q = hA(T_s - T_\infty)} \quad (W)$$

$$q'' = h(T_s - T_\infty) \quad (W/m^2)$$

h = convective heat transfer coefficient ($W/m^2 \cdot K$)

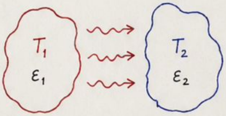
T_s = surface temperature (K)

T_∞ = fluid bulk temperature (K)

Note: h depends on fluid properties, flow condition, and geometry.

③ Radiation

Heat transfer by electromagnetic waves. Does not require a medium.



Stefan–Boltzmann Law:

$$\boxed{q = \varepsilon \sigma A (T_s^4 - T_{sur}^4)} \quad (W)$$

σ = Stefan–Boltzmann constant = 5.67×10^{-8} $W/m^2 \cdot K^4$

ε = emissivity ($0 \le \varepsilon \le 1$)

A = area (m^2)

T_s = surface temperature (K)

T_{sur} = surrounding temperature (K)

For two large, diffuse-gray surfaces facing each other:

$$\boxed{q = \frac{\sigma A (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}} \quad (W)$$

Overall Heat Transfer

(Combination of modes)

$$T_1 \xrightarrow{R_1} \xrightarrow{R_2} \cdots \xrightarrow{R_n} T_2$$

$$\boxed{q = \frac{T_1 - T_2}{R_{total}}} \quad (W) \qquad R_{total} = \sum R_i$$

R = Thermal resistance (K/W)

Common Thermal Resistances:

- Conduction (plane wall): $R_{cond} = \frac{L}{kA}$
- Convection (surface): $R_{conv} = \frac{1}{hA}$
- Radiation (surface to large surroundings, linearized):

  $R_{rad} = \frac{1}{h_r A}$, where $h_r = 4\varepsilon\sigma T_m^3$

  T_m = mean absolute temperature (K)

Key Points

- Heat flows from high T to low T.
- Conduction: through solids or stationary media.
- Convection: between surface and moving fluid.
- Radiation: by electromagnetic waves.
- Units of heat transfer rate: W (or W/m^2 for heat flux).

Remember: 1 K = 1 °C (temperature difference)

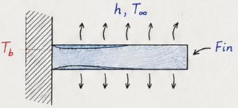
Always check: mode, geometry, properties, and boundary conditions! ☆

Extended Surfaces (Fins) – Summary

Extended surfaces (fins) increase heat transfer between a surface and a fluid by increasing the surface area.

1) What is a Fin?

A fin (extended surface) is a projection from a surface that increases heat transfer to the surrounding fluid by increasing the area available.



2) Fin Efficiency (η_f)

Effectiveness of a fin compared to the same area at the base temperature T_b.

$$\boxed{\eta_f = \frac{\dot{q}_{fin}}{h A_f (T_b - T_\infty)}}$$

A_f = total surface area of fin (including sides and tip)

$$0 < \eta_f \le 1$$

3) Fin Effectiveness (ε_f)

How much a fin increases heat transfer compared to no fin.

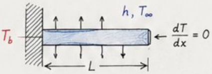
$$\boxed{\varepsilon_f = \frac{\dot{q}_{fin}}{h A_b (T_b - T_\infty)}}$$

A_b = base area of the fin

$\varepsilon_f > 1$ (beneficial fin)

4) Common Fin Types (One-Dimensional, Constant Cross-Section)

(a) Straight fin, insulated tip



$$\boxed{m = \sqrt{\frac{hP}{kA_c}}}$$

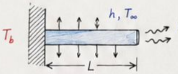
$$\boxed{\dot{q}_{fin} = \sqrt{hPkA_c}\,(T_b - T_\infty)\tanh(mL)}$$

$$\boxed{\eta_f = \frac{\tanh(mL)}{mL}}$$

P = perimeter, A_c = cross-sectional area

k = thermal conductivity

(b) Straight fin, convective tip

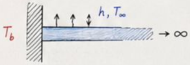


$$\boxed{m = \sqrt{\frac{hP}{kA_c}}}$$

$$\boxed{\dot{q}_{fin} = \sqrt{hPkA_c}\,(T_b - T_\infty)\frac{\sinh(mL) + \frac{h}{mk}\cosh(mL)}{\cosh(mL) + \frac{h}{mk}\sinh(mL)}}$$

$$\boxed{\eta_f = \frac{\dot{q}_{fin}}{h A_f (T_b - T_\infty)}}$$

(c) Straight fin, infinite length



$$\boxed{m = \sqrt{\frac{hP}{kA_c}}}$$

$$\boxed{\dot{q}_{fin} = \sqrt{hPkA_c}\,(T_b - T_\infty)}$$

$$\boxed{\eta_f = \frac{1}{mL}}$$ (when $L \to \infty$, $\eta_f \to 0$)

Notes:
- Fins are most effective when k is high, h is high, and the fin is long.
- Adding many fins increases total heat transfer, even if each fin has low efficiency.

Total heat transfer with fins:

$$\dot{Q}_{total} = h A_b (T_b - T_\infty) + \sum \dot{q}_{fin}$$

External Convective Heat Transfer – Summary

Heat transfer between a solid surface and a moving fluid (air or liquid) over the surface.

1) Key Relation

$$\boxed{\dot{Q} = h A_s (T_s - T_\infty)}$$

h = convective heat transfer coefficient ($W/m^2 \cdot K$)

A_s = surface area (m^2)

T_s = surface temperature (K)

T_∞ = free-stream fluid temperature (K)

2) Important Dimensionless Numbers

Reynolds number: $Re_L = \dfrac{\rho V_\infty L}{\mu}$ (inertia / viscous forces)

Prandtl number: $Pr = \dfrac{c_p \mu}{k}$ (momentum / thermal diffusivity)

Nusselt number: $Nu_L = \dfrac{hL}{k}$ (dimensionless heat transfer)

3) Common Correlations (Over Flat Plate)

(a) Laminar flow ($Re_L < 5 \times 10^5$)

- Local Nusselt number:

$$\boxed{Nu_x = 0.332\, Re_x^{1/2}\, Pr^{1/3}} \Rightarrow h_x = \frac{k}{x} Nu_x$$

- Average over length L:

$$\boxed{\overline{Nu}_L = 0.664\, Re_L^{1/2}\, Pr^{1/3}} \Rightarrow \overline{h}_L = \frac{k}{L}\overline{Nu}_L$$

(b) Turbulent flow ($Re_L > 5 \times 10^5$)

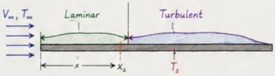
- Average over length L (including transition):

$$\boxed{\overline{Nu}_L = 0.037\, Re_L^{4/5}\, Pr^{1/3}} \Rightarrow \overline{h}_L = \frac{k}{L}\overline{Nu}_L$$

V_∞ = free-stream velocity, L = characteristic length (plate length in flow direction)

Correlations depend on geometry, flow regime, surface roughness, and fluid properties.

4) Flow Regimes over a Flat Plate



Transition location

$Re_{x_c} \approx 5 \times 10^5$

5) Factors Affecting h

- Fluid properties (k, μ, ρ, c_p)
- Flow velocity (higher $V_\infty \Rightarrow$ higher h)
- Surface geometry and size (L)
- Flow regime (laminar < turbulent)
- Surface roughness

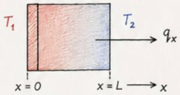
6) Typical h Values (Air, 1 atm)

- Natural convection: 2 – 25 $W/m^2 \cdot K$
- Forced convection (air):
  - Low velocity (1–5 m/s): 10 – 50
  - High velocity (10–30 m/s): 50 – 200
- Gases: 5 – 200 $W/m^2 \cdot K$
- Liquids (water): 100 – 10,000+

One-Dimensional Heat Transfer by Conduction – Summary

Steady-state heat transfer through a homogeneous wall with constant thermal conductivity and no internal heat generation.

1) Plane Wall



Governing Equation

$$\frac{d^2T}{dx^2} = 0$$

Temperature Distribution

$$T(x) = T_1 + \frac{T_2 - T_1}{L}\, x$$

Heat Flux (Fourier's Law)

$$q_x = -k\frac{dT}{dx} = k\,\frac{T_1 - T_2}{L} \quad (\text{W/m}^2)$$

Heat Transfer Rate

$$\dot{Q} = q_x A = kA\,\frac{T_1 - T_2}{L} \quad (\text{W})$$

Thermal Resistance Form

$$\dot{Q} = \frac{T_1 - T_2}{R_{cond}}, \quad R_{cond} = \frac{L}{kA} \quad (\text{K/W})$$

where

- k = thermal conductivity (W/m·K)
- A = cross-sectional area (m^2)
- L = thickness (m)

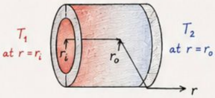
Boundary Conditions

$$T(0) = T_1, \quad T(L) = T_2$$

Key Points

- Temperature varies linearly across the wall.
- Heat flux is constant throughout the wall.
- Thicker wall (large L) → smaller $\dot{Q}$.
- Higher k or larger A → larger $\dot{Q}$.

2) Cylindrical Wall



Governing Equation (Steady State)

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

Temperature Distribution

$$T(r) = T_1 + \frac{T_2 - T_1}{\ln\left(\frac{r_o}{r_i}\right)}\ln\left(\frac{r}{r_i}\right)$$

Heat Flux (Radial Direction)

$$q_r = -k\frac{dT}{dr} = \frac{k(T_1 - T_2)}{r\ln\left(\frac{r_o}{r_i}\right)} \quad (\text{W/m}^2)$$

Heat Transfer Rate

$$\dot{Q} = 2\pi k L\,\frac{T_1 - T_2}{\ln\left(\frac{r_o}{r_i}\right)} \quad (\text{W})$$

Thermal Resistance Form

$$\dot{Q} = \frac{T_1 - T_2}{R_{cond}}, \quad R_{cond} = \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi k L} \quad (\text{K/W})$$

where

- r_i = inner radius (m)
- r_o = outer radius (m)
- L = length of cylinder (m)
- k = thermal conductivity (W/m·K)

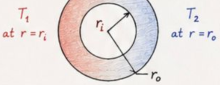
Boundary Conditions

$$T(r_i) = T_1, \quad T(r_o) = T_2$$

Key Points

- Temperature varies logarithmically with r.
- Heat flux decreases as r increases (inversely proportional to r).
- $\dot{Q}$ increases with length L and k.
- Larger radius ratio (r_o/r_i) → larger thermal resistance.

3) Spherical Wall



Governing Equation (Steady State)

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$$

Temperature Distribution

$$T(r) = T_1 + (T_2 - T_1)\frac{\left(\frac{1}{r_i} - \frac{1}{r}\right)}{\left(\frac{1}{r_i} - \frac{1}{r_o}\right)}$$

Heat Flux (Radial Direction)

$$q_r = -k\frac{dT}{dr} = \frac{k(T_1 - T_2)}{r^2\left(\frac{1}{r_i} - \frac{1}{r_o}\right)} \quad (\text{W/m}^2)$$

Heat Transfer Rate

$$\dot{Q} = 4\pi k\,\frac{r_i r_o (T_1 - T_2)}{r_o - r_i} \quad (\text{W})$$

Thermal Resistance Form

$$\dot{Q} = \frac{T_1 - T_2}{R_{cond}}, \quad R_{cond} = \frac{r_o - r_i}{4\pi k r_i r_o} \quad (\text{K/W})$$

where

- r_i = inner radius (m)
- r_o = outer radius (m)
- k = thermal conductivity (W/m·K)

Boundary Conditions

$$T(r_i) = T_1, \quad T(r_o) = T_2$$

Key Points

- Temperature varies with $1/r$.
- Heat flux decreases with $1/r^2$.
- $\dot{Q}$ increases with k and with product $r_i r_o$.
- Smaller wall thickness $(r_o - r_i)$ → smaller thermal resistance.

Comparison of Key Results

Geometry	Temperature Distribution	Heat Transfer Rate $\dot{Q}$	Thermal Resistance R_{cond}	Heat Flux Variation
Plane wall	Linear in x	$kA\,\frac{T_1 - T_2}{L}$	$\frac{L}{kA}$	Constant
Cylindrical wall	Logarithmic in r	$2\pi k L\,\frac{T_1 - T_2}{\ln(r_o/r_i)}$	$\frac{\ln(r_o/r_i)}{2\pi k L}$	$\propto \frac{1}{r}$
Spherical wall	Varies with $1/r$	$4\pi k\,\frac{r_i r_o (T_1 - T_2)}{r_o - r_i}$	$\frac{r_o - r_i}{4\pi k r_i r_o}$	$\propto \frac{1}{r^2}$

General Assumptions

- Steady-state
- One-dimensional conduction
- Homogeneous material (constant k)
- No internal heat generation
- Perfect thermal contact at surfaces

Remember: Identify geometry first → choose correct equation → apply boundary conditions → compute $\dot{Q}$ or T. ☆

Internal Convective Heat Transfer – Summary

Heat transfer between a fluid and the internal surface of a duct, pipe, or channel.

1) Key Relation

$$\boxed{\dot{Q} = h_i A_s (T_s - T_b)}$$

h_i = internal convective heat transfer coefficient ($W/m^2 \cdot K$)
A_s = internal surface area (m^2)
T_s = wall (surface) temperature (K)
T_b = bulk mean fluid temperature (K)

Bulk mean temperature:

$$T_b = \frac{\int_A uT\,dA}{\int_A u\,dA}$$

(u = local velocity)

2) Dimensionless Numbers

Reynolds number:

$$\boxed{Re = \frac{\rho V D_h}{\mu}}$$

D_h = hydraulic diameter

Prandtl number:

$$\boxed{Pr = \frac{c_p \mu}{k}}$$

Nusselt number (based on D_h):

$$\boxed{Nu = \frac{h_i D_h}{k}}$$

D_h = hydraulic diameter = $\frac{4A_c}{P}$
A_c = flow area, P = wetted perimeter
k = thermal conductivity of fluid ($W/m \cdot K$)

3) Flow Regimes

Laminar flow: $Re < 2300$
Transition: $2300 \le Re \le 10^4$
Turbulent flow: $Re > 10^4$

4) Important Correlations

(a) Laminar flow

- Fully developed, constant wall temperature:
  - Circular tube: $Nu = 3.66$
  - Non-circular duct (high aspect ratio): $Nu = 4.36$
- Fully developed, constant heat flux:
  - Circular tube: $Nu = 4.36$
- Entrance region (laminar):

$$Nu_x = 1.86 \left(Re\ Pr\ \frac{D_h}{x}\right)^{1/3}$$

(b) Turbulent flow (Dittus – Boelter equation)

$$\boxed{Nu = 0.023\ Re^{0.8}\ Pr^n}$$

$n = 0.4$ (heating)
$n = 0.3$ (cooling)

(c) Turbulent flow (Gnielinski equation)

$$\boxed{Nu = \frac{(f/8)\,(Re - 1000)\,Pr}{1 + 12.7\,(f/8)^{1/2}\,(Pr^{2/3} - 1)}}$$

f = Darcy friction factor (from Moody chart)

5) Hydraulic Diameter for Common Ducts

Circular tube | Rectangular duct | Annulus | Non-circular (general)

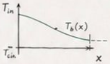
$D_h = D$ | $D_h = \frac{2ab}{a+b}$ | $D_h = D_o - D_i$ | $D_h = \frac{4A_c}{P}$

6) Heat Transfer Coefficient

$$h_i = \frac{Nu\,k}{D_h}$$

Use appropriate correlation to find Nu, then compute h_i.

7) Bulk Temperature Behavior



Bulk temperature changes along the flow direction due to heat transfer.

Heat Exchangers – Summary

Devices used to transfer heat between two fluids at different temperatures.
Fluids are separated by a solid wall (direct contact is avoided).

1) Basic Energy Balance

Steady state (no heat loss):

$$\boxed{\dot{Q}_h = \dot{Q}_c = \dot{Q}}$$

$$\dot{Q}_h = \dot{m}_h c_{p,h} (T_{h,in} - T_{h,out})$$
$$\dot{Q}_c = \dot{m}_c c_{p,c} (T_{c,out} - T_{c,in})$$

2) Overall Heat Transfer Coefficient

$$\dot{Q} = UA\,\Delta T_{lm}$$

U = overall heat transfer coefficient ($W/m^2 \cdot K$)
A = heat transfer surface area (m^2)
ΔT_{lm} = log mean temperature difference (K)

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$$
$$\Delta T_1 = T_{h,in} - T_{c,out}$$
$$\Delta T_2 = T_{h,out} - T_{c,in}$$

3) Flow Arrangements

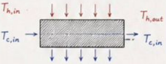
(a) Parallel flow

LMTD:
$\Delta T_1 = T_{h,in} - T_{c,in}$
$\Delta T_2 = T_{h,out} - T_{c,out}$

(b) Counter flow

LMTD:
$\Delta T_1 = T_{h,in} - T_{c,out}$
$\Delta T_2 = T_{h,out} - T_{c,in}$

(c) Cross flow (one fluid unmixed)



Use correction factor:
$\Delta T_{lm,actual} = F \cdot \Delta T_{lm,counter}$
$0 < F \le 1$ (from charts)

4) Effectiveness – NTU Method (any configuration)

Effectiveness: $\boxed{\varepsilon = \frac{\dot{Q}_{actual}}{\dot{Q}_{max}}}$

Number of transfer units: $\boxed{NTU = \frac{UA}{C_{min}}}$

Capacity rate: $\boxed{C = \dot{m} c_p}$

$C_{min} = \min(C_h, C_c)$, $C_{max} = \max(C_h, C_c)$, $C_r = \frac{C_{min}}{C_{max}}$

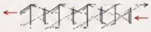
From charts/tables: $\varepsilon = f(NTU, C_r, \text{configuration})$
Then $\dot{Q}_{actual} = \varepsilon\,\dot{Q}_{max}$, $\dot{Q}_{max} = C_{min}(T_{h,in} - T_{c,in})$

5) Common Heat Exchanger Types

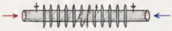
- Double pipe (concentric)

- Shell and tube

- Plate heat exchanger



- Finned tube



6) Design Steps (Summary)

1. Specify knowns and required $\dot{Q}$.
2. Choose exchanger type & arrangement.
3. Estimate U, calculate A using $\dot{Q} = UA\Delta T_{lm}$ or NTU-effectiveness method.
4. Check pressure drop and adjust if needed.
5. Finalize design.

Remember: Choose proper correlations, correct properties (at bulk temperature), and check flow regime! ☆